Sir R.A. Fisher, "The Underworld of Probability," Sankhya, Vol 18, Sept 1957 pp 201-210

Uncertainty of Rank A: random variable, known distribution.
Uncertainty of Rank B: percentile points of the distribution of x are themselves random variables, with known distribution.
Uncertainty of Rank C: percentile points of the distribution of the percentile points of the distribution of x are random variables, with known distribution.
etc. A priori and a posteriori probs in Bayes' formula are statements with uncertainty of rank B.

Behavioral implications: given uncertainty of rank A, a gambler will bet on the outcome of an event. Given uncertainty of rank B (10% of the dice in a box from which a die has been chosen at random will throw an Ace more than 1/5 of the time) a gambler can bet on the long-run proportions, e.g., of an Ace; or bet that another gambler, with a given decision rule, will lose in the long run. cA third gambler, who knows the distribution of the distribution of p(x) in a consignment of boxes of dice, "may know with confidence what odds he can profitably accept in betting against the success of B's wager." 203 (

"Right probs may be increasingly ambiguous

"A firm of contractors building a bridge will employ engineers who use tables of the strength of materials, and perhaps, empirical data on the frequency of destructive floods, hurricanes or earthquakes, of a strength capable of jeopardising their work. In view of recognised uncertainties of the strength of the structure, and its parts, they build in what are called "factors of safety" at great expense, to cover, as they hope, all reasonable risks. Nevertheless, the corporation operating such a structure will

will usually seek out an Insurance Office, who at an appropriate premium, or in other words at chosen odds, will underwrite the residual risk of the accidental destruction of the structure. They are in effect betting that for an assigned period the precautions of the engineers will be shown to have been sufficient." 203

"Moreover, even the simple investor risking his savings by buying ordinary stock, perhaps in an Insurance Company, is in effect laying down a stake on the proposition that the company will collect enough in premiums to cover the large risks it has underwritten, and above this to pay a reasonable dividend on their ordinary shares."

"The mathematical underworld I have introduced has a qualitative resemblance to the specification of risk in quite familiar operations in our own world. It is only the mathematical tradition that has been too shy of uncertainty, too firmly attached to the ethereal regions of mathematical certainty, to have attempted any very deep exploration." 203

((SUBJECTS IN MY URN EXAMPLE WHO ASSIGNED EQUAL LIKELIHOODS TO ALL DISTRIBUTIONS BETWEEN (1/3, 2/3,0) and (1/3, 0, 2/3) WOULD BE SUBJECT OF TO UNCERTAINTY OF RANK B. BUT, FACED WITH SYMMETRIC UNCERTAINTY ABOUT THESE DISTRIBUTIONS THEY MIGHT FOLLOW SOME OTHER CRITERION, SUCH AS MINIMAX. At some rank level, uncertainties are bound to become symmetric; question then of assigning equal probabilities or of consulting payoffs and expectations. Fisher's own statements about requiring favorable odds or risk premiums, or assigning "factors of safety" at great expense to cover "all reasonable risks" suggest that in doing so, the engineers and the insurance company

will almost surely be violating the Savage axioms, i.e., contradicting the assumption that they are assigning probabilities to prob distributions. (or if they are, these probabilities are being influenced by the payoffs for each individual action, and are differing for each action; hence cannot be inferred as probabilities for a given set of events from observing a pattern of bets). CONSTRUCT EXAMPLE BASED ON "SAFETY FACTORS," where "success" or "failure" reverse meaning in different situations. E.G., I WOULD RATHER BUY A FAMILIAR STOCK THAN ANOTHER, UNFAMILIAR ONE, AND I WOULD RATHER SELL IT SHORT--without having the beliefs about the different variabilities of these two stocks that might rationalise this.

((Do people ask themselves: (1) What is my "best guess" about the probability of this event? 2) what probability do I assign to this estimate; 3) what probability do I assign to this estimate of this estimate? etc. or do they ask: (1) what is my "best" guess about the probability; 2) what probability do I assign to this guess? 3) what is the "worst reasonable guess?" 4) what are some "good reasonable guesses"? etc. ))

Imagine 2 wires, one with a definite prob. of melting at a given temperature, the other with an ambiguous prob (some "best guess"). You might use first, both to conduct heat/energy, and to serve as a fuse. (Or, using second, you might use big "safety factor" in either case; "as if" — for given size — it were "much more likely" to melt, and "much less likely.")